ON BOUNDARY PROBLEM FOR NONLINEAR PARABOLIC EQUATIONS WITH LEVY LAPLACIAN

Kovtun I.I.

National University of Life and Environmental Sciences of Ukraine,
Learning-scientific institute of energetic and automatic,
Department higher and applied mathematic,
15, Geroiv Oborony, Kyiv, 03041, Ukraine
e-mail: ira@otblesk.com

Let H be a separable real Hilbert space. Let $\overline{\Omega} = \Omega \cup \Gamma = \left\{ x \in H : \|x\|_H^2 \le R^2 \right\}$ be the ball in H. Let $U(t,x)$ be the function in $[0,\infty)\times\Omega$, and $\Delta_L U(t,x)$ be the Levy Laplacian [1], [2].

Consider the boundary value problem for nonlinear equations with Levy Laplacian

$$\frac{\partial U(t,x)}{\partial t} = f(\Delta_L U(t,x)) \quad \text{in} \quad \Omega, \quad U(t,x)\big|_\Gamma = G(t,x), \tag{1}$$

where $f(\xi)$ is a given continuous twice differentiable function. The equation $f(\varsigma) = z$ can be solved with respect to $\xi : \xi = \varphi(z)$. $G(t,x)$ is a given function,

The solution of problem (1) exist, when exist solution $V(t,x)$ of boundary problem for the heat equation $\dfrac{\partial V(\tau,x)}{\partial t} = \Delta_L V(\tau,x)$ in Ω, $V(t,x)\big|_\Gamma = G(t,x)$.

Theorem, Let the equation $f'\left(\varphi\left(\left.\dfrac{\partial V(\tau,x)}{\partial \tau}\right|_{\tau = X + T(x)}\right)\right)[t - X] - T(x) = 0$ can be solved with respect to $X = \chi(t,x)$, and $\chi(t,x)\big|_\Gamma = t$, $T(x) = \dfrac{1}{2}\left(R^2 - \|x\|_H^2\right)$. Then the solution boundary problem (1) is

$$U(t,x) = f\left(\psi(\chi(t,x))\right)[t - \chi(t,x)] - \psi(\chi(t,x))T(x) + V\left(\chi(t,x) + T(x), x\right),$$

where $\psi(\chi(t,x)) = \varphi\left(\left.\dfrac{\partial V(\tau,x)}{\partial \tau}\right|_{\tau = \chi(t,x) + T(x)}\right)$.

References.

1. *Levy P.* Problemes concrets d'analyse fonctionnelle. - Paris: G.-V. 1951. 510 p.
2. *Feller M.N.* The Levy Laplacian. -Cambridge etc.: Cambridge Univ. Press. 2005. 153 p.