GALOIS GROUPS FROM LIE SYMMETRIES VIEWPOINT

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Galois groups for algebraic equations are discussed in the framework of Lie's continuous transformation groups. First we construct a Lie symmetry group of a given algebraic equation. Then we restrict the action of the group to the roots of the equation in question. The result will be the Galois group. The approach is illustrated by several examples. Among them is the equation

$$x^2 + 1 = 0. \tag{1}$$

The calculation shows that Equation (1) has the symmetry group composed by the transformations

$$T_\alpha : \quad \bar{x} = \frac{x+\alpha}{1-\alpha x}, \tag{2}$$

$$S_\beta : \quad \bar{x} = \frac{x+\beta}{\beta x - 1} \tag{3}$$

with arbitrary parameters α and β. The restriction of the transformations T_α and S_β on the roots $x_1 = i, \quad x_2 = -i$ of Equation (1) gives

$$T_\alpha(x_1) = x_1, \quad T_\alpha(x_2) = x_2,$$

$$S_\beta(x_1) = x_2, \quad S_\beta(x_2) = x_1.$$

Hence, the Galois group is composed by the identity transformation of roots and by the permutation (x_1, x_2) of the roots. Formally it is written

$$\{1, \ (x_1, x_2)\}.$$

This approach was first proposed in [1] (see also [2]). In our talk we present a simple way to understand this approach by providing more detailed calculations and new examples. We hope that our work will help to comprehend the concept of the Galois group so that it could be used in teaching mathematics and introduced to mathematical curricula even at the level of high schools. Interested students can easily continue studying this subject in the modern terminology in terms of extensions of fields etc.

References.

1. *Ibragimov N.H.* Primer of group analysis., Znanie, No. 8, Moscow, 1989, 44 p. (In Russian).
2. *Ibragimov N.H.* A bridge between Lie symmetries and Galois groups., In *"Differential Equations: Geometry, Symmetries and Integrability."The Abel Symposium 2008. Proc. of Fifth Abel Symposium, Tromsö, Norway, June 17-22, 2008.* Eds. B. Kruglikov, V. Lychagin, E. Straume (2009), Springer, 159-172.