ON THE ERROR ESTIMATE OF APPROXIMATION OF FUNCTIONS OF BOUNDED VARIATION BY SZASZ-MIRAKYAN OPERATORS

Singh S. N.

SKM University, Dumka at Jamtara College, Jamtara, Jharkhand-815351 (India).

The Szasz-Mirakyan operators play an important role in the theory of approximation. They have been studied intensively in connection with different branches of analysis. The Szasz-Mirakyan operator is defined as

$$S_n(f, x) = \sum_{k=0}^{\infty} f(k/n)\, p_k(nx), \text{ where}$$

$$p_k(nx) = e^{-nx}(nx)^k/k!, \qquad n \in N,\ x \in R_0.$$

The Szasz-Mirakyan operators S_n are defined in terms of a sample of given function f on the points k/n, for $k \in N_0$, $n \in N$. Many research papers [3, 4, 5] appear with certain modifications in this operator $S_n(f, x)$.

Grof [1] proved that if f be continuous on $[0, \infty)$ and $f(x) = O(e^{\alpha x})$, for some $\alpha > 0$, as $x \to \infty$ then for all $A > 0$ and $x \in [0, A]$

$$S_n(f, x) - f(x) = O\,(\,\omega_{2A}(f, n^{-1/2})),$$

where

$$\omega_A(f, \delta) = \sup \{|\, f(x+t) - f(x)| : |\, t| \le \delta\}.$$

This result was further improved by Hermann. He proved that the above result holds if $f(t)=O(t^{\alpha t})$, $\alpha > 0$. Cheng [1] estimated the rate of convergence of $S_n(f, x)$. He proved that if f be continuous function of bounded variation on every finite subinterval of $[0, \infty)$ and $f(t) = O(t^{\alpha t})$ for some $\alpha > 0$ as $t \to \infty$, then if $x \in (0, \infty)$ is irrational, then for n sufficiently large,

$$|S_n(f, x) - (1/2)[f(x+0)+f(x-0)]| \le ((3+x)/nx) \sum_{k=1}^{n} V_{x-x/\sqrt{k}}^{x+x/\sqrt{k}} (g_x) + O(x^{-1/2}n^{1/2})\, |f(x+) - f(x-)|$$

$$+ O(1)\, (4x)^{4\alpha x}\, (nx)^{-1/2}(e/4)^{nx},$$

where $V_a^b(g)$ is the total variation of g on [a, b], and $g_x(t) = f(t) - f(x+0)$, $x < t < \infty$; $= 0$ if $t = x$; $= f(t) - f(x-0)$ if $0 \le t < x$. We shall also consider the continuous functions of bounded variation defined on $[0, \infty)$ and find the error estimate of approximation by Szasz-Mirakyan operators maintaining its original form with a different approach, also a better estimate of approximation has been obtained in this paper.

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