BOUNDARY VALUE PROBLEM FOR QUASILINEAR PARABOLIC EQUATIONS WITH A LEVY LAPLACIAN

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Let H be a real infinite dimensional Hilbert space. Let a scalar function F depend on H is twice strongly differentiable at a point x_0. The Lévy Laplacian of F at the point x_0 is defined the formula [1]

$$\Delta_L F(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} (F''(x_0) f_k, f_k)_H,$$

where $F''(x)$ is the Hessian of $F(x)$, and $\{f_k\}_1^\infty$ is an orthonormal basis in H.

Let Ω be a bounded domain in the Hilbert space H (that is a bounded open set in H), and $\overline{\Omega} = \Omega \cup \Gamma$ be a domain in H with boundary Γ :

$$\Omega = \{x \in H : 0 \leq Q(x) \langle R^2\}, \quad \Gamma = \{x \in H : Q(x) = R^2\},$$

where $Q(x)$ is a twice strongly differentiable function such that $\Delta_L Q(x) = \gamma,\ \gamma\rangle 0$ is a positive constant.

Consider the Cauchy problem

$$\frac{\partial U(t,x)}{\partial t} = \Delta_L U(t,x) + f_0(U(t,x)), \qquad U(0,x) = U_0(x), \tag{1}$$

where $U(t,x)$ is a function on $[0,\mathfrak{T}] \times H$, $f_0(\xi)$ is a given function of one variable, $U_0(x)$ is a given function defined on H.

Assume exists a primitive $\varphi(\xi) = \int \frac{d\xi}{f_0(\xi)}$ and the inverse function φ^{-1}. Assume exists a solution of the Cauchy problem for the heat equation

$$\frac{\partial V(t,x)}{\partial t} = \Delta_L V(t,x), \quad V(0,x) = U_0(x).$$

Then the solution $U(t,x)$ of the Cauchy problem (1) is

$$U(t,x) = \varphi^{-1}(t + \varphi(V(t,x))).$$

References.

1. *Lévy P.* Sur la generalisation de léquation de Laplace dans domaine fonctionnelle. *C.R.Acad. Sc.* **168**, 1919. P. 752-755.