NOISE EQUILIBRIUM IN LINEAR APPROXIMATION

Krivosheev O.I.

ICS RAS, Profsoyuznaya 65, Moscow, 450075, Russia, 8-926-147-77-36,
E-mail: o-krivosheev@yandex.ru

We discuss a very simple and may be the simplest possible or minimal model that simultaneously describes price volatilities and credit leverages assumed approximately constant at time trajectory (as a financial strategy goal parameter), that is chosen to maximize investor's capital return rate at given volatility and at given mean physical capital return rate. We consider a typical Hardin's common pool resource (further CPL) tragedy problem, at that arises from the fact that everybody tries to rise its leverage until global disequilibrium situation in the economic system arises and further up to the situation in that bankruptcy risk prevents further leverage rising because expected capital return rate is maximal and begins to diminish.

There is possible direct approach, when we solve three simultaneous vector ODE equations 1) for debt, 2) for price (fast dynamical equation), 3) for physical capital. Still (if we only can in some way estimate an effective IRR fall-down shock period T) a purely analytical calculation based on a set of two derivatives matrices and (optionally) one additional capital reflow matrix can estimate the approximate leverage and approximate simultaneous price fall-dawn vector, that either defines return rate fall-down depth.

At that approach we use a two step procedure. First we calculate a sensitivity matrix $J = \frac{\partial \vec{i}}{\partial \vec{p}} = \left(\frac{\partial i_{k1}}{\partial p_{k2}} \right)$, where $\vec{i}\,(\vec{p}, \vec{p}_K)$ is a physical capital return rate, p is a current price vector $\vec{p}_K$ - is a (mean) historical capital creation prices, that can be estimated by its long term equilibrium meaning $\vec{p}_K{}^{Eqv}$, $C(\vec{K},\vec{l}\,) = \frac{\partial \Delta \vec{Q}(\vec{K},\vec{l}, \vec{p}_K{}^{Eqv}, \vec{p})}{\partial \vec{p}}$, where $\Delta\vec{Q}(..., \vec{p})$ is a demand-supply difference.

If the growth rates of every technology is equal, than we have no capital reflows, end the reflow matrix can be (mutually) omitted. So we get three rather natural equations: the scalar stability boarder $Max\,\mathrm{Re}\,Spec\ C(\vec{l}\,) = 0$, obvious equation for the main eigen price deviation vector $\Delta\vec{p}$: $C(\vec{l}\,)\Delta\vec{p} = 0$, and a series of price to leverage conversion equations based on $\Delta\vec{i} \approx J\Delta\vec{p}$ or $\Delta\vec{i} \approx \frac{d\vec{i}}{d\vec{p}}\Delta\vec{p}$, and $[\vec{l}_c]\Delta\vec{i} = \vec{d}$, where $[\vec{\eta}]$ is a diagonal matrix with $\vec{\eta}$ vector on it. The last one is inspired by the $l = \frac{\vec{d}}{\Delta\vec{i}} k_T = \vec{l}_c k_T$, where $k_T = 1/(1 - \exp(-dT(\varepsilon)))$ is the only coefficient, responsible for the ODE-solution complexity. Where $\varepsilon(l) = Max\,\mathrm{Re}\,Spec\,C(l)$ - is a small overcriticity of real solution, with $\vec{l} = \vec{l}_c k_T$. But the situation may be simpler, if T -period is well-known for the given economy or due to the regulation approach.

So three-four equations $Max\,\mathrm{Re}\,Spec\ C(\vec{l}\,) = 0$, $[\vec{l}_c]J\Delta\vec{p} = \vec{d}$ and $C(\vec{l}\,)\Delta\vec{p} = \vec{0}$ with $\vec{l} = \vec{l}_c/(1 - \exp(-dT))$ completely define financial leverages, return rate $\Delta\vec{i}\,(\Delta\vec{p})$ and price $\Delta\vec{p}$ volatilities