

PERIODIC STRUCTURES ON PHYLLOTACTIC PATTERNS

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The purpose of the work is to show the possibility of displaying some periodic and quasi-periodic sequences in the pattern of the alternate leaf arrangement (phyllotaxis). Our approach proposes, in particular, the distribution of the spectrum of a real number on a polar integer lattice. This coordinate system will be conditionally designated as phyllotactic. A graph in this coordinate grid is considered as a tool for visualizing and studying periodic phenomena, whether it be the movement of celestial bodies or the laws of musical harmony. In the field of view lie such mathematical concepts as the spectrum of a real number, Beatty sequences inclusive, continued fractions, Euclid's algorithm, the projection of a two-dimensional lattice onto a one-dimensional space, the Bresenham's line. The spectrum of a real number α is defined as an infinite set of integers. $\text{Spec}(\alpha) = \{[\alpha], [2\alpha], [3\alpha], \dots\}; [x] =$ largest integer less than or equal to x (floor). The concept of the spectrum of a real number, which is closely related to Bresenham's "midpoint algorithm" and Euclid's algorithm, finds its application in calendar calculations. Computer animation of some calendar systems in phyllotactic representation is proposed. The spectra of the numbers $12/7$ and $12/5$ reflect the alternation of white and black colors of the piano keys, and their projection onto the phyllotactic pattern with a divergence angle $360^\circ * 5/12 = 150^\circ$ reverses the 2D arrangement of the 1D circle of fifths. Some calendars employ two cycles running simultaneously. We offer a computer "phyllotactic" animation of the Eastern calendar of the 60-year cycle, the Maya Tzolkin calendar with a 260-day cycle, the 52-year circle of the calendar, called Xihuitl.

The projection of integer sequences onto the phyllotactic lattice is in line with the concept of cognitive graphics proposed by Alexander Zenkin. "Cognitive visualization is aimed at presenting the essence of a scientific abstract problem area, i.e. the most fundamental connections and relationships between the elements of this area, in a graphic form in order to see and discover fundamentally new knowledge of a conceptual nature" [1].

References

1. Zenkin, A., (2010). *Cognitive (Semantic) Visualization of the Continuum Problem and Mirror Symmetric Proofs in the Transfinite Numbers Theory*
<https://vismath1.tripod.com/zen/zen1.htm>.