ERROR BOUNDS FOR RECOVERY OF ELLIPTIC DIFFERENTIAL OPERATORS WITH CONSTANT COEFFICIENTS ON NONISOTROPIC BESOV-NIKOL'SKII CLASSES USING SPECTRUM INFORMATION

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Let us consider the problem of optimal linear recovery for elliptic differential operator with constant coefficients on non - isotropic Nikol'skii-Besov spaces $B^s_{p\,\theta}(\mathbb{R}^n)$ using spectrum information (information on Fourier transform) in L_q — norm.

Namely, let

$$\mathcal{L} := \sum_{|\alpha|\le m} a_\alpha \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}}$$

be elliptic differential operator with constant coefficients; let $B^s_{p\,\theta}(\mathbb{R}^n)$ be Nikol'skii-Besov space and L_q be the space of measurable function on $\mathbb{R}^n$ with standard norm $\|\cdot\|_q$.

Denote Fourier transform of distribution $f \in S'(\mathbb{R}^n)$ by $\mathcal{F}(f)$. For $f \in S'(\mathbb{R}^n)$ denote restriction of $\mathcal{F}(f)$ on $\Omega_\sigma := \{\xi : \|\xi\|_{\boldsymbol{a}} < \sigma\} \subset \mathbb{R}^n$ by $\mathcal{F}|_{\Omega_\sigma}$. We use $\mathcal{F}(f)|_{\Omega_\sigma}$ as information on function $f \in B^s_{p\,\theta}(\mathbb{R}^n)$.

Then the problem of recovery is to estimate the quantity

$$E(B^s_{p\,\theta}(\mathbb{R}^n), \mathcal{L}, \mathcal{F}|_{\Omega_\sigma}, L_q) := \inf_S \sup_{\|f|B^s_{p\,\theta}(\mathbb{R}^n)\|\le 1} \|\mathcal{L}f - \mathcal{L}S[\mathcal{F}(f)|_{\Omega_\sigma}]\|_q$$

where inf is taken over all linear methods $S : \mathcal{F}(B^s_{p\,\theta})|_{\Omega_\sigma} \to L_q$, and to find linear methods $\tilde{S}$ for which the order of the quantity is realized.

We obtained order exact estimate for the quantity $E(B^s_{p\,\theta}(\mathbb{R}^n), \mathcal{L}, \mathcal{F}|_{\Omega_\sigma}, L_q)$ and construct corresponding optimal linear method as action of the differentiation operator on special "partial" sum of the expansion of function f with respect to Meyer-David system of orthonormal multivariate wavelets.