MULTI-OBJECTIVE OPTIMIZATION. REAL-TIME PARETO NAVIGATION

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In multi-objective optimization compromises between the functions always have to be made. So (in case of minimization problem) the value of one of them can not be decreased without increasing the value of at least one another at the same time. Hence in multi-criteria optimization a person who has to take the responsibility for decision making is required.

Consider the sliders model, where the certain slider corresponds to each objective. When user moves one of the sliders, he(she) is able to see how the values of other objectives are changing. Suppose there is a database containing some Pareto-optimal precalculated solutions. Suggested approach for Pareto navigation is based on the one described in [1]:

$$\min\{z \in \mathbb{R} |\, (\boldsymbol{Yv})_i - \boldsymbol{y}_i^R + \boldsymbol{s}_i = z, i \in \mathcal{K} \setminus \{j\}, (\boldsymbol{Yv})_j = \tau, \boldsymbol{Yv} \leq \boldsymbol{b}, \sum_{i=1}^{m} \boldsymbol{v}_i = 1, \boldsymbol{s} \geq 0\}.$$

With some modifications like using not all the solutions from the database but only points from the previous solution's and target's neighbourhood this method gives very good results in case of convex functions. For non-convex functions some problems might occur, namely the interpolated solution may become infeasible.

For non-convex case the method based on Pascoletti-Serafini approach [2] was used. The non-linear problem can be formulated in the following way:

$$\min_{\boldsymbol{x} \in X, \boldsymbol{s}, z} z, \quad s.t.\ \boldsymbol{y}^R + z\boldsymbol{v} = \ \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{s},\ \ \boldsymbol{s} \geq 0$$

where $\boldsymbol{y}^R$ and $\boldsymbol{v}$ are parameters (reference point and dominated direction correspondingly), z (step length) and $\boldsymbol{s}$ (slack vector) are auxiliary variables. But this method is good for example if there are simple formulas to calculate all the necessary functions. But sometimes by functions is meant simulation which takes quite a long time.

An application with graphical user interface combining both methods was developed in order to give to the planner the opportunity to choose one depending on the problem. This work was performed at Optimization department of Fraunhofer Institute for Industrial Mathematics(ITWM) joint with Technical University Kaiserslautern, Germany under supervision of Dr. Anton Winterfeld.

References.

1. *M.Monz, K.H.Kuefer, T.R.Bortfeld and C.Thieke*. Pareto navigation-algorithmic foundation of interactive multi-criteria IMRT planning — Physics In Medicine And Biology, Jan.2008. Pp. 985-998
2. *A.Pascoletti, P.Serafini*. Differential Conditions for Constrained Nonlinear Programming via Pareto Optimization — Optimization Theory And Applications, July 2007. Pp. 399-411