CENTRALLY SYMMETRIC STEADY STATES IN A MODEL OF ELECTRODIFFUSION

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We consider a mathematical model of electrodiffusion in a centrally symmetric case [1]-[2]. This model describes in particular the transport of the Li^+ ions inside the graphite spherical particles in the porous negative electrodes [3] due to diffusion and migration:

$$\frac{\partial c}{\partial t}(t,r) = D\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial c}{\partial r} + \frac{zF}{RT}c(t,r)\frac{\partial u}{\partial r}\right)\right]; \quad \frac{\partial c}{\partial r}(t,0) = 0; \quad c(t,R) = C^*;$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] = -\frac{F}{\varepsilon_0\varepsilon}c(t,r); \quad \frac{\partial u}{\partial r}(t,0) = 0; \quad u(t,R) = U^*. \tag{1}$$

Here *c(t,r)* is the Li^+ ion concentration, and *u(t,r)* is the electric potential. We prove that this model possesses the unique steady state solution *c=C(r), u=U(r)*:

$$C(r) = \frac{2\widetilde{C}}{(1-\gamma\cdot\widetilde{C}r^2)^2}; U(r) = \widetilde{U} + \frac{2}{\alpha}\cdot\ln\left(1-\gamma\cdot Cr^2\right); \alpha = \frac{zF}{RT}, \beta = \frac{zF}{\varepsilon_0\varepsilon}, \gamma = \frac{\alpha\cdot\beta}{4}. \tag{2}$$

The constants $\widetilde{C}, \widetilde{U}$ are the roots of certain quadratic equations; for each equation only one of its roots is eligible. We study numerically the behavior of time-dependent solutions to (1) with various initial conditions and demonstrate that the spatially non-uniform steady state (2) is the stable attractor for the time-dependent solutions to (1) regardless of the initial distributions of ion concentration and electric potential.

References.

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