

THE PROCESS OF ALGORITHM CONSTRUCTING OF NONLINEAR POLYNOMIAL FIRST INTEGRALS

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It is known that there is no general method of integrating nonlinear systems of ordinary differential equations. In this paper, we propose a new computational algorithm for finding the nonlinear polynomial first integrals for ordinary differential equations (ODEs) with polynomial right hand member [1]. The algorithm is based upon the necessary and sufficient condition of the existence of first integrals [2].

Consider a system of ODE:

$$\dot{x}_i = P_i(x_1, x_2, \dots, x_n) \quad (1)$$

where $P_i(x_1, x_2, \dots, x_n)$ is a general case of polynomial of arbitrary degree

For system (1) the first integral takes the following form:

$$W(x_1, x_2, \dots, x_n) = C \quad (2)$$

where $W(x_1, x_2, \dots, x_n)$ is a polynomial with undetermined coefficients which are to be found.

To find the unknown factors in the expression (2), we write the following necessary and sufficient condition for the existence of first integrals (2) for (1):

$$\sum_{i=1}^n \frac{\partial W}{\partial x_i} \cdot P_i(x_1, x_2, \dots, x_n) = 0 \quad (3)$$

Next, on the left side of (3), we select the coefficients of equal powers and equate them to zero. As a result, we get a system of linear algebraic equations for the unknown coefficients of the polynomial $W(x_1, x_2, \dots, x_n)$. We solve the system of linear algebraic equations for the unknown coefficients and write the resulting first integral in the form of (2) where the coefficients found are already known.

Literature:

1. *Shakurov I.R., Asadullin R.M.* The process of algorithm constructing of first integrals for systems of ordinary differential equations (ODEs) with polynomial right hand sides // Scientific Bashkortostan: Almanac, № 2 - Ufa: Vagant, 2009. Pp. 3-14.
2. *Erugin N.P.* Reading book for the general course of differential equations. - Minsk: Science and Technology, 1979. 570 p