

GLOBAL BIFURCATION ANALYSIS OF A GENERALIZED LOTKA–VOLTERRA SYSTEM

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We study a quartic dynamical system which models the dynamics of the populations of predators and their prey that use the group defense strategy in a given ecological, epidemiological or immunological system and which is a variation on the classical Lotka–Volterra system:

$$\begin{aligned}\dot{x} &= x((1 - \lambda x)(\alpha x^2 + \beta x + 1) - y) \equiv P, \\ \dot{y} &= -y((\delta + \mu y)(\alpha x^2 + \beta x + 1) - x) \equiv Q,\end{aligned}\tag{1}$$

where $\alpha \geq 0$, $\delta > 0$, $\lambda > 0$, $\mu \geq 0$ and $\beta > -2\sqrt{\alpha}$ are parameters. Such a quartic dynamical model was studied earlier, for instance, in [1, 2]. However, its qualitative analysis was incomplete, since the global bifurcations of limit cycles could not be studied properly by means of the methods and techniques which were used earlier in the qualitative theory of dynamical systems.

Together with (1), we also consider an auxiliary system

$$\dot{x} = P - \gamma Q, \quad \dot{y} = Q + \gamma P,\tag{2}$$

applying to these systems new bifurcation methods and geometric approaches developed in [3] and completing the qualitative analysis of system (1).

In particular, using (2), we prove the following theorem (see [4]).

Theorem 1. *System (1) has at most two limit cycles.*

Besides, we discuss how to use higher-dimensional Lotka–Volterra systems as biomedical or ecological models.

References.

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